

FP3 Moch Kerime 2

1. $4 \tanh^2 x - 2 \operatorname{sech}^2 x = 3$

$\therefore 4 - 4 \operatorname{sech}^2 x = 2 \operatorname{sech}^2 x = 3$

$\therefore 4 - 6 \operatorname{sech}^2 x = 3$

$6 \operatorname{sech}^2 x = 1$

$\Rightarrow \operatorname{sech}^2 x = \frac{1}{6}$

$\Rightarrow \cosh^2 x = 6$

$\cosh x > 0$

$\Rightarrow \cosh x = \sqrt{6}$

$\Rightarrow \operatorname{arcosh} \sqrt{6} = \ln \left(\sqrt{6} + \sqrt{6+1} \right)$

$\cosh(x) \equiv \cosh(\pm x) \Rightarrow x = \pm \ln \left(\sqrt{6} + \sqrt{5} \right)$

$$2. \quad y = 2 \cosh\left(\frac{1}{2}x\right)$$

$$c^2 - s^2 = 1$$

$$\frac{\partial y}{\partial x} = \sinh\left(\frac{1}{2}x\right)$$

$$\text{Surface Area} = 2\pi \int_{-\ln 2}^{\ln 2} 2 \cosh\left(\frac{x}{2}\right) \sqrt{1 + \sinh^2\left(\frac{x}{2}\right)} \, dx$$

$$= 4\pi \int_{-\ln 2}^{\ln 2} \cosh^2\left(\frac{x}{2}\right) \, dx$$

$$\begin{aligned} \cosh 2x &= c^2 + s^2 \\ &= c^2 + c^2 - 1 \\ \cosh 2x &= 2c^2 - 1 \end{aligned}$$

$$= \frac{4\pi}{2} \int_{-\ln 2}^{\ln 2} \cosh x + 1 \, dx$$

$$\begin{aligned} \cosh^2 x &= \frac{(\cosh 2x + 1)}{2} \\ \therefore \cosh^2\left(\frac{x}{2}\right) &= \frac{(\cosh x + 1)}{2} \end{aligned}$$

$$= 2\pi \left[\sinh x + x \right]_{-\ln 2}^{\ln 2}$$

$$= 2\pi \left(\frac{e^{\ln 2} - e^{-\ln 2}}{2} + \ln 2 - \frac{e^{-\ln 2} - e^{\ln 2}}{2} + \ln 2 \right)$$

~~$$= 2\pi \left(\frac{3}{4} + 2\ln 2 - \frac{3}{4} \right) = 2\pi \left(\frac{3}{2} + 2\ln 2 \right)$$~~

~~$$= 4\pi \ln 2 \text{ units}^2$$~~

$$= 3\pi + 4\pi \ln 2$$

$$3. \quad x = \frac{3}{\sinh \theta} = 3(\sinh \theta)^{-1}$$

$$\begin{aligned} \frac{\partial x}{\partial \theta} &= -3(\sinh \theta)^{-2} \cosh \theta \\ &= -\frac{3 \cosh \theta}{\sinh^2 \theta} \end{aligned}$$

$$\therefore \partial x = -\frac{3 \cosh \theta}{\sinh^2 \theta} \partial \theta$$

$$\begin{aligned} \frac{1}{x\sqrt{x^2+9}} &= \frac{1}{\frac{3}{\sinh \theta} \sqrt{\frac{9}{\sinh^2 \theta} + 9}} \\ &= \frac{1}{\frac{9}{\sinh \theta} \sqrt{\frac{1}{\sinh^2 \theta} + 1}} \end{aligned}$$

$$\therefore \frac{1}{x\sqrt{x^2+9}} = \frac{\sinh \theta}{9 \sqrt{\frac{1}{\sinh^2 \theta} + 1}}$$

$$\begin{aligned} \therefore 3\sqrt{3} &= \frac{3}{\sinh \theta} \Rightarrow \sinh \theta = \frac{1}{\sqrt{3}} \\ &\Rightarrow \theta = \ln \sqrt{3} \end{aligned}$$

$$4 = \frac{3}{\sinh \theta} \Rightarrow \sinh \theta = \frac{3}{4} \Rightarrow \theta = \ln 2$$

$$\therefore \int_{\ln 2}^{\ln \sqrt{3}} \frac{\sinh \theta}{9 \sqrt{\sinh^2 \theta + 1}} \cdot -\frac{3 \cosh \theta}{\sinh^2 \theta} d\theta$$

$$= - \int_{\ln 2}^{\ln \sqrt{3}} \frac{3 \cosh \theta}{9 \sinh \theta \sqrt{\sinh^2 \theta + 1}} d\theta$$

$$= - \int_{\ln 2}^{\ln \sqrt{3}} \frac{3 \cosh \theta}{9 \sqrt{1 + \sinh^2 \theta}} d\theta$$

$$= - \int_{\ln 2}^{\ln \sqrt{3}} \frac{3}{9} \frac{\cosh \theta}{\sqrt{\cosh^2 \theta}} d\theta$$

$$= - \int_{\ln 2}^{\ln \sqrt{3}} \frac{1}{3} d\theta = - \left[\frac{1}{3} \theta \right]_{\ln 2}^{\ln \sqrt{3}}$$

$$= - \left(\frac{1}{3} \ln \sqrt{3} - \frac{1}{3} \ln 2 \right)$$

$$= \frac{1}{3} \ln 2 - \frac{1}{3} \ln \sqrt{3} = \frac{1}{3} \ln \left(\frac{2}{\sqrt{3}} \right)$$

$$= \frac{1}{3} \ln \left(\frac{2}{\sqrt{3}} \right) = \frac{1}{3} \ln \sqrt{\frac{4}{3}}$$

$$= \frac{1}{3} \ln \sqrt{\frac{4}{3}} = \frac{1}{6} \ln \left(\frac{4}{3} \right)$$

4 (a) $y = \arctan \sqrt{x}$

$\sin^2 + \cos^2 = 1$
 $\tan^2 + 1 = \sec^2$

$\therefore \tan y = \sqrt{x}$

$\therefore \frac{\partial y}{\partial x} \sec^2 y = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}} \quad \tan^2 y + 1 = x + 1$

$\therefore \frac{\partial y}{\partial x} (\tan^2 y + 1) = \frac{1}{2\sqrt{x}}$

$\therefore \frac{\partial y}{\partial x} = \frac{1}{2\sqrt{x}} \cdot \frac{1}{x+1}$

$\frac{\partial y}{\partial x} = \frac{1}{2\sqrt{x}(x+1)}$

$\therefore \left(\frac{\partial y}{\partial x}\right)_{x=1/4} = \frac{1}{2\sqrt{1/4} (1/4+1)} = \frac{4}{5}$

(b) $\frac{\partial y}{\partial x} = (2\sqrt{x}(x+1))^{-1} \quad \frac{1}{2} x^{-1/2}$

$\therefore \frac{\partial^2 y}{\partial x^2} = - (2\sqrt{x}(x+1))^{-2} \left[2\sqrt{x} (1) + (x+1) \left(2 \cdot \frac{1}{2} x^{-1/2} \right) \right]$

$= - (2\sqrt{x}(x+1))^{-2} \left[2\sqrt{x} + \frac{(x+1)}{\sqrt{x}} \right]$

$= - \frac{2\sqrt{x} + \frac{(x+1)}{\sqrt{x}}}{(2\sqrt{x}(x+1))^2} = - \frac{3x+1}{4\sqrt{x} \cdot x (x+1)^2}$

$$\frac{\partial^2 y}{\partial n^2} = - \frac{3x+1}{4x^{3/2}(x+1)^2}$$

$$\begin{aligned}\therefore 2x(1+x) \frac{\partial^2 y}{\partial n^2} &= - \frac{3x+1}{4x^{3/2}(x+1)^2} \cdot 2x(1+x) \\ &= - \frac{2x(3x+1)}{4x^{3/2}(x+1)} = - \frac{2(3x+1)}{4\sqrt{x}(x+1)} \\ &= - \frac{3x+1}{2\sqrt{x}(x+1)}\end{aligned}$$

$$+ (1+3x) \frac{\partial y}{\partial n} = \frac{1}{2\sqrt{x}(x+1)}$$

$$\therefore (1+3x) \frac{\partial y}{\partial n} = \frac{1+3x}{2\sqrt{x}(x+1)}$$

$$\therefore 2x(1+x) \frac{\partial^2 y}{\partial n^2} + (1+3x) \frac{\partial y}{\partial n} = - \frac{2(3x+1)}{4\sqrt{x}(x+1)} + \frac{1+3x}{2\sqrt{x}(x+1)}$$

$$= - \frac{(3x+1)}{2\sqrt{x}(x+1)} + \frac{1+3x}{2\sqrt{x}(x+1)}$$

$$= \frac{-(3x+1) + 1+3x}{2\sqrt{x}(x+1)} = \frac{0}{2\sqrt{x}(x+1)}$$

$$= 0$$

as required.

5
Q). $I_n = \int_0^{\pi/2} \sin^n x \, dx$

$u = \sin^n x$ $u' = n \sin^{n-1} x \cos x$
 $v' = 1$ $v = x$

5 Q). $I_n = \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \sin^{n-1} x \sin x \, dx$

$u = \sin^{n-1} x$ $u' = (n-1) \sin^{n-2} x \cos x$
 $v' = \sin x$ $v = -\cos x$

$\therefore I_n = \left[-\cos x \sin^{n-1} x \right]_0^{\pi/2} + (n-1) \int_0^{\pi/2} \sin^{n-2} x \cos^2 x \, dx$

$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x (1 - \sin^2 x) \, dx$

$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x - \sin^n x \, dx$

$\therefore I_n = (n-1) \int_0^{\pi/2} \sin^{n-2} x - (n-1) \int_0^{\pi/2} \sin^n x \, dx$

$\therefore I_n = (n-1) I_{n-2} - (n-1) I_n$

$\therefore I_n + (n-1) I_n = (n-1) I_{n-2}$

$\therefore n I_n = (n-1) I_{n-2}$

$$\therefore I_{n-2} = \frac{n-1}{n} I_{n-2} \text{ as required.}$$

(6) $\int_0^{\pi/2} x \sin^5 x \cos x \, dx$

Let $u = x$ $u' = 1$

$v' = \cos x (\sin x)^5$ $v = \frac{\sin^6 x}{6}$

$$\therefore \int_0^{\pi/2} x \sin^5 x \cos x \, dx = \left[\frac{x \sin^6 x}{6} \right]_0^{\pi/2} - \frac{1}{6} I_6$$

$$= \frac{\pi}{12} - \frac{1}{6} I_6$$

$$= \frac{\pi}{12} - \frac{1}{6} \cdot \frac{5}{6} I_4$$

$$= \frac{\pi}{12} - \frac{5}{36} \cdot \frac{3}{4} I_2$$

$$= \frac{\pi}{12} - \frac{5}{48} \cdot \frac{1}{2} I_0$$

$$= \frac{\pi}{12} - \frac{5}{96} \int_0^{\pi/2} 1 \, dx$$

$$= \frac{\pi}{12} - \frac{5}{96} [x]_0^{\pi/2} = \frac{\pi}{12} - \frac{5}{96} \left(\frac{\pi}{2} \right)$$

$$= \frac{\pi}{12} - \frac{5}{96} \left(\frac{6\pi}{12} \right)$$

~~$$= \frac{91}{96} \cdot \frac{\pi}{12}$$~~

$$= \frac{11}{192} \pi$$

~~$$= \frac{91}{1152} \pi$$~~

$$6(a). \vec{PQ} = \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ -4 \end{pmatrix}$$

$$\vec{QR} = \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$$

$$\vec{PQ} \times \vec{QR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & -4 \\ 2 & 2 & -2 \end{vmatrix} = \begin{vmatrix} -3 & 4 & -4 \\ 2 & 2 & -2 \end{vmatrix}$$

$$= \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix}$$

$$(b) \hat{n} = \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix}$$

$$\underline{Q} \cdot \hat{n} = \underline{P} \cdot \hat{n}$$

$$\therefore -14y - 14z = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -14 \\ -14 \end{pmatrix} = 42 - 14$$

$$\therefore 14y + 14z = -28$$

$$\underline{y + z = -2}$$

$$(C). \quad \vec{r} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 6$$

$$\vec{r} \cdot \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} = 28$$

$$\left. \begin{array}{l} \vec{r} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 6 \\ \vec{r} \cdot \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} = 28 \end{array} \right\} \begin{array}{l} x+y-z=6 \\ y+z=-2 \end{array}$$

$$\therefore y = -2 - z$$

$$x + (-2 - z) - z = 6 \Rightarrow x - 2 - 2z = 6$$

$$x - 2 - 2z = 6$$

$$x = 8 + 2z$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8+2z \\ -2-z \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} + z \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$z = \lambda$$

$$\Rightarrow \vec{r} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$$

~~$$\vec{r} \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$~~

$$\vec{a} = \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\therefore \left(\vec{r} - \begin{pmatrix} 8 \\ -2 \\ 0 \end{pmatrix} \right) \times \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0$$

$$7(a). \quad Ax = \begin{pmatrix} 2 & k & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 18+3k \\ 12 \\ -8 \end{pmatrix} \quad 5-2$$

$$= \begin{pmatrix} 3(6+k) \\ 3(4) \\ 3(-2) \end{pmatrix} = \begin{pmatrix} 9 \\ 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2(6+k) \\ 7(3) \\ 4(-2) \end{pmatrix} = \begin{pmatrix} 4\left(\frac{9}{2} + \frac{3}{4}k\right) \\ 4(3) \\ 4(-2) \end{pmatrix} = 4 \begin{pmatrix} \frac{9}{2} + \frac{3}{4}k \\ 3 \\ -2 \end{pmatrix}$$

$$\Rightarrow \frac{9}{2} + \frac{3}{4}k = 9 \quad \Rightarrow k = \left(9 - \frac{9}{2}\right) \times \frac{4}{3}$$

$$\therefore k = \underline{\underline{6}} \text{ as required.}$$

(b) $\lambda = 4$ is an eigen value

$$A - \lambda I = \begin{pmatrix} 2-\lambda & 6 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = (2-\lambda)(1-\lambda)^2 - 6(1-\lambda)$$

$$= (1-\lambda)(2-\lambda)(1-\lambda) - 6(1-\lambda)$$

$$= (1-\lambda)(2 - 3\lambda + \lambda^2 - 6)$$

$$= (1-\lambda)(\lambda^2 - 3\lambda - 4)$$

$$= (1-\lambda)(\lambda - 4)(\lambda + 1)$$

\therefore $\lambda = 1$ $\lambda = 4$ are eigen-values
 $\lambda = -1$

$$\textcircled{C) } AP = \begin{pmatrix} 2 & 6 & 0 \\ 1 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} t-2 \\ t \\ 2t \end{pmatrix}$$

$$= \begin{pmatrix} 8t-4 \\ 2t-2 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 8 \\ 2 \\ 0 \end{pmatrix}$$

∴ the line represented has equation

$$\underline{r} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 8 \\ 2 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{aligned} x &= 8t - 4 = \cancel{2t-2} \\ y &= 2t - 2 \\ z &= 0 \end{aligned} \quad \Rightarrow \begin{aligned} y + 2 &= 2t \\ \cancel{2y + 4} &= \\ 4y + 8 &= 8t \end{aligned}$$

$$\therefore x = 8t - 4 = 4y + 8 - 4 = 4y + 4$$

$$\therefore x = 4y + 4$$

$$\Rightarrow \underline{\underline{x - 4y - 4 = 0}}$$

$$8(a). \quad \frac{x^2}{25} - \frac{y^2}{9} = 1$$

$$\frac{2}{25}x - \frac{2}{9}y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{25} \frac{x}{y} \quad \frac{dy}{dx} = \frac{9}{25} \frac{x}{y}$$

$$\therefore \left(\frac{dy}{dx} \right) \text{ at } p = \frac{9}{25} \cdot \frac{5}{3} \frac{\sec u}{\tan u} = \frac{3}{5} \operatorname{cosec} u$$

$$\therefore y - 3 \tan u = \frac{3}{5} (x - 5 \sec u) \operatorname{cosec} u$$

$$\therefore 5y - 15 \tan u = 3 \operatorname{cosec} u (x - 5 \sec u)$$

$$\therefore 5y - 15 \tan u = 3 \operatorname{cosec} u x - 15 \operatorname{cosec} u \sec u$$

$$\cancel{\operatorname{cosec} u} \times \sin u \quad 5y \sin u - \frac{15 \sin^2 u}{\cos u} = 3x - \frac{15}{\cos u}$$

$$\therefore \cancel{5y \sin u} \quad 3x = 5y \sin u - \frac{15 \sin^2 u}{\cos u} + \frac{15}{\cos u}$$

$$\therefore 3x = 5y \sin u + \frac{15(1 - \sin^2 u)}{\cos u}$$

since $1 - \sin^2 u = \cos^2 u$

$$\therefore 3x = 5y \sin u + 15 \cos u \quad \text{as required}$$

(b) at R $y = \frac{3}{5}x$

$\Rightarrow 3x = 5 \left(\frac{3}{5}x\right) \sin u + 15 \cos u$

$\therefore 3x = 3x \sin u + 15 \cos u$

$\therefore \cancel{x} \rightarrow x (1 - \sin u)(3x) = 15 \cos u$

$\therefore x = \frac{5 \cos u}{1 - \sin u} \quad \& \quad y = \frac{3}{5} \left(\frac{5 \cos u}{1 - \sin u}\right) = \frac{3 \cos u}{1 - \sin u}$

$\therefore R \left(\frac{5 \cos u}{1 - \sin u}, \frac{3 \cos u}{1 - \sin u} \right)$

at S, $y = -\frac{3}{5}x$

$\therefore \cancel{3x} \quad 3x = 5 \left(-\frac{3}{5}x\right) \sin u + 15 \cos u$

$\therefore 3x = -3x \sin u + 15 \cos u$

~~$y = \frac{3}{5} \left(\frac{5 \cos u}{1 + \sin u}\right)$~~

$\therefore (1 + \sin u)3x = 15 \cos u$

$\therefore x = \frac{5 \cos u}{1 + \sin u} \quad y = \frac{-3 \cos u}{1 + \sin u}$

$$\therefore \text{at } \int \left(\frac{5 \cos u}{1 + \sin u}, \frac{-3 \cos u}{1 + \sin u} \right)$$

~~$\therefore P$~~ x coordinate of

$$x_p = \frac{1}{2} \left(\frac{5 \cos u}{1 + \sin u} + \frac{5 \cos u}{1 - \sin u} \right)$$

$$x_p = \frac{1}{2} \left(\frac{5 \cos u (1 - \sin u) + 5 \cos u (1 + \sin u)}{1 - \sin^2 u} \right)$$

$$\therefore x_p = \frac{1}{2} \left(\frac{5 \cos u (1 - \sin u) + 5 \cos u (1 + \sin u)}{\cos^2 u} \right)$$

$$\therefore x_p = \frac{1}{2} \left(\frac{5 \cos u (2)}{\cos^2 u} \right) = \frac{5 \cos u}{\cos^2 u} = \frac{5}{\cos u}$$

$$\therefore x_p = \frac{5}{\cos u} = 5 \sec u$$

(b) at R $y = \frac{3}{5}x$

$$\Rightarrow 3x = 5 \left(\frac{3}{5}x \right) \sin u + 5 \cos u$$

$$\therefore 3x = 3x \sin u + 5 \cos u$$

$$\therefore \cancel{x = x} (1 - \sin u)(3x) = 5 \cos u$$

$$\therefore x = \frac{5 \cos u}{1 - \sin u} \quad \& \quad y = \frac{3}{5} \left(\frac{5 \cos u}{1 - \sin u} \right) = \frac{3 \cos u}{1 - \sin u}$$

$$\therefore R \left(\frac{5 \cos u}{1 - \sin u}, \frac{3 \cos u}{1 - \sin u} \right)$$

at S, $y = -\frac{3}{5}x$

$$\therefore \cancel{3x} \quad 3x = 5 \left(-\frac{3}{5}x \right) \sin u + 5 \cos u$$

$$\therefore 3x = -3x \sin u + 5 \cos u$$

$$\cancel{y = -\frac{3}{5} \left(\frac{5 \cos u}{1 + \sin u} \right)}$$

$$\therefore (1 + \sin u)3x = 5 \cos u$$

$$\therefore x = \frac{5 \cos u}{1 + \sin u} \quad y = \frac{-3 \cos u}{1 + \sin u}$$

$$y_p = \frac{1}{2} \left(\frac{3 \cos u}{1 - \sin u} - \frac{3 \cos u}{1 + \sin u} \right)$$

$$= \frac{1}{2} \left(\frac{3 \cos u (1 + \sin u)}{1 - \sin^2 u} - \frac{3 \cos u (1 - \sin u)}{1 - \sin^2 u} \right)$$

$$= \frac{1}{2} \left(\frac{3 \cos u (1 + \sin u - (1 - \sin u))}{\cos^2 u} \right)$$

$$= \frac{1}{2} \left(\frac{3 (2 \sin u)}{\cos u} \right) = \frac{3 \sin u}{\cos u}$$

$$= \underline{\underline{3 \tan u}} \text{ as required}$$

$$x_p = 5 \sec u \quad y_p = 3 \tan u$$

\therefore P is midpoint of RS